**Chapter 11**

**Inferences About Population Variances**

**Learning Objectives**

1. Understand the importance of variance in a decision–making situation.

2 Understand the role of statistical inference in developing conclusions about the variance of a single population.

3. Know that the sampling distribution of (*n* – 1) *s*2/*σ* 2 has a chi–square distribution and be able to use this result to develop a confidence interval estimate of *σ* 2.

4. Be able to compute *p*–values using the chi–square distribution.

5. Know how to test hypotheses involving *σ* 2.

6. Understand the role of statistical inference in developing conclusions about two population variances.

7. Know that the sampling distribution of  has an *F* distribution and be able to use this result to test hypotheses involving two population variances.

8. Be able to compute *p*–values using the *F* distribution.

**Solutions:**

1. a. 11.070

b. 27.488

c. 9.591

d. 23.209

e. 9.390

2. *s*2 = 25

a. With 19 degrees of freedom = 30.144 and = 10.117



15.76  *σ* 2  46.95

b. With 19 degrees of freedom  = 32.852 and  = 8.907



14.46  *σ* 2  53.33

c. 3.8  *σ*  7.3

3. 

Degrees of Freedom = (16 – 1) = 15

Using table, *p*–value is between .025 and .05

Exact *p*–value using=27.08 is .0281

*p*–value  .05, reject *H*0

Critical value approach

 = 24.996

Reject *H*0 if 24.996

27.08 > 24.996, reject *H*0

4. a. *n* = 18

*s*2 = .36

 = 27.587

= 8.672 (17 degrees of freedom)



.22  *σ* 2  .71

b. .47  **  .84

5. a.  or $1,330,000 per year

b. 

 or about $1 million

b. With *df* = n – 1 = 10 – 1 = 9  = 19.023 = 2.700



.48 *σ* 2  3.35

c. .69 ** 1.83

6. a. 

The sample mean amount spent on a Halloween costume was $41.

b. 



c. With *df*  = (n – 1) = 15  = 27.488 = 6.262





301.88  *σ* 2  1325.14

17.37 ** 36.40

7. a. 

The sample mean quarterly total return for General Electric is 3.2%. This is the estimate of the population mean percent total return per quarter for General Electric.

b. 



c. With *df*  = (n – 1) = 7  = 16.013 = 1.690





110.76  *σ* 2  1049.47

d. 10.52 ** 32.40

8. a. 



b. 

c. 11 degrees of freedom

 = 21.920 = 3.816



.2383  *σ* 2  1.3687

.4882  ** 1.1699

9. *H*0: *σ* 2 .0004

*H*a: *σ* 2 .0004



Degrees of freedom = *n* – 1 = 29

Using table, *p*–value is greater than .10

Exact *p*–value using= 36.25 is .1664

*p*–value > .05, do not reject *H*0. The product specification does not appear to be violated.

10. a. **

b. **

c*.* 

d. Hypothesis for **= 12 is for *σ* 2 = (12)2 = 144

*H*0: *σ* 2  144

*H*a: *σ* 2 144



Degrees of freedom = *n* – 1 = 14

Usingtable, *p*–value is greater than 2(1 – .90) = .20

Exact *p*–value corresponding to= 11.54 is 2(1 – .6431) = .7138

*p*–value > .05, do not reject *H*0. The hypothesis that the population standard deviation is 12

cannot be rejected

11. a. 





b. *H*0: *σ* 2 =.70

*H*a: *σ* 2 ≠.70

c. 

Degrees of freedom = *n* – 1 = 11

Usingtable, area in upper tail is greater than .10

Two–tail *p*–value is greater than .20

Exact two–tailed *p*–value corresponding to= 10.84 is 2(.4568) = .9136

*p*–value > .05, do not reject *H*0. We cannot conclude the variance in bond yields has changed.

12. a. 

b. *H*0: *σ* 2 .94

*H*a: *σ* 2 ≠.94



Degrees of freedom = *n* – 1 = 11

Usingtable, area in tail is greater than .10

Two–tail *p*–value is greater than .20

Exact *p*–value corresponding to= 9.49 is .8465

*p*–value > .05, cannot reject *H*0.

13. a. *F*.05 = 3.33

b. *F*.025 = 2.76

c. *F*.01 = 4.50

d. 

14. a. 

Degrees of freedom 15 and 20

Using *F* table, *p*–value is between .025 and .05

Exact *p*–value corresponding to *F* = 2.4 is .0345

*p*–value  .05, reject *H*0. Conclude 

b. *F*.05 = 2.20

Reject *H*0 if *F*  2.20

2.4  2.20, reject *H*0. Conclude 

15. a. Larger sample variance is 



Degrees of freedom 20 and 25

Using *F* table, area in tail is between .025 and .05

Two–tail *p*–value is between .05 and .10

Exact *p*–value corresponding to *F* = 2.05 is .0904

*p*–value > .05, do not reject *H*0.

b. Since we have a two–tailed test



Reject *H*0 if *F*  2.30

2.05 < 2.30, do not reject *H*0

16. For this type of hypothesis test, we place the larger variance in the numerator. So the Fidelity variance is given the subscript of 1.







Degrees of freedom in the numerator and denominator are both 59

Using the *F* table, *p*–value is between .05 and .025

Exact *p*–value corresponding to *F* = 1.59 is .0387

*p*–value  .05, reject *H*0. We conclude that the Fidelity fund has a greater variance than the American Century fund.

17. a. Population 1 is 4 year old automobiles





b. 

Degrees of freedom 25 and 24

Using *F* table, *p*–value is less than .01

Exact *p*–value corresponding to *F* = 2.89 is .0057

*p*–value  .01, reject *H*0. Conclude that 4 year old automobiles have a larger variance in annual repair costs compared to 2 year old automobiles. This is expected due to the fact that older automobiles are more likely to have some more expensive repairs which lead to greater variance in the annual repair costs.

18. We place the larger sample variance in the numerator. So, the Merrill Lynch variance is given the subscript of 1.







Degrees of freedom 15 and 9

Using *F* table, area in tail is greater than .10

Two–tail *p*–value is greater than .20

Exact *p*–value corresponding to *F* = 1.44 is .5906

*p*–value > .10, do not reject *H*0. We cannot conclude there is a statistically significant difference between the variances for the two companies.

19. 



.0489

.0059



Degrees of freedom 24 and 21

Using *F* table, area in tail is less than .01

Two–tail *p*–value is less than .02

Exact *p*–value  0

*p*–value .05, reject *H*0. The process variances are significantly different. Machine 1 offers the best opportunity for process quality improvements.

Note that the sample means are similar with the mean bag weights of approximately 3.3 grams. However, the process variances are significantly different.

20. 





Degrees of freedom 25 and 24

Using *F* table, area in tail is less than .01

Two–tail *p*–value is less than .02

Exact *p*–value  0

*p*–value  .05, reject H0. The population variances are not equal for seniors and managers.

21. a. Consider the talk time use as population 1 and Internet use as population 2.





b.  hours



 hours



c. 

Degrees of freedom  and 

The *p*–value is the upper–tail area at *F* = 6.00.

From the *F* table, the *p*–value is less than .01

Exact *p*–value corresponding to *F* = 6.00 is .006

*p*–value ≤ .05, reject *H*0. The population variance in battery hours of use for the talk time application is larger than the population variance in battery hours of use for the Internet application.

22. a. Population 1 – Wet pavement.







Degrees of freedom 15 and 15

Using *F* table, *p*–value is less than .01

Exact *p*–value corresponding to *F* = 4.00 is .0054

*p*–value .05, reject *H*0. Conclude that there is greater variability in stopping distances on wet pavement.

b. Drive carefully on wet pavement because of the variability in stopping distances.

23. a. *s*2 = (30) 2 = 900

b.  = 30.144 and  = 10.117 (19 degrees of freedom)



567  ** 2  1690

c. 23.8  **  41.1

24. With 12 degrees of freedom,

= 23.337 = 4.404



114.9  ** 2  609

10.72  **  24.68

25. a. 

b. 



c.  = 32.852 = 8.907 (19 degrees of freedom)



2890  ** 2  10,659

53.76  **  103.24

26. a. *H*0: *σ* 2 .0001

*H*a: *σ* 2 .0001



Degrees of freedom = *n* – 1 = 14

Usingtable, *p*– value is between .01 and .025

Exact *p*–value corresponding to= 27.44 is .0169

*p*–value  .10, reject *H*0. Variance exceeds maximum variance requirement.

b.  = 23.685

= 6.571 (14 degrees of freedom)



.00012  ** 2  .00042

27. *H*0: *σ* 2 .02

*H*a: *σ* 2 .02



Degrees of freedom = *n* – 1 = 40

Usingtable, *p*– value is greater than .10

Exact *p*–value corresponding to= 51.20 is .1104

*p*–value > .05, do not reject *H*0. The population variance does not appear to be exceeding the standard.

28. *H*0: *σ* 2 

*H*a: *σ* 2 



Degrees of freedom = *n* – 1 = 21

Usingtable, *p*–value is between .05 and .10

Exact *p*–value corresponding to= 31.50 is .0657

*p*–value  .10, reject *H*0. Conclude that ** 2 > 1.

29. 

*H*0: ** 2 = 10

*H*a: ** 2  10



Degrees of freedom = *n* – 1 = 8

Usingtable, area in tail is greater than .10

Two–tail *p*– value is greater than .20

Exact *p*–value corresponding to= 10.16 is .5080

*p*–value > .10, do not reject *H*0

30. a. Try *n* = 15

= 26.119

= 5.629 (14 degrees of freedom)



34.3  ** 2  159.2

5.86  **  12.62

A sample size of 15 was used.

b. *n* = 25; expect the width of the interval to be smaller.

 = 39.364

 = 12.401 (24 degrees of freedom)



39.02  ** 2  126.86

6.25  **  11.13

31. 



Population 1 is women’s scores.



Degrees of freedom 19 and 29

Using Excel of Minitab, we find the exact two–tail *p*–value corresponding to *F* = 1.24 is .5876

*p*–value > .10, do not reject . There is not a statistically significant difference in the variances. We cannot conclude that there is a difference in the variability of golf scores for male and female professional golfers.

32. 



Use critical value approach since *F* tables do not have 351 and 72 degrees of freedom.

*F*.025 = 1.47

Reject *H*0 if *F*  1.47



*F* < 1.47, do not reject *H*0. We are not able to conclude students who complete the course and students who drop out have different variances of grade point averages.

33. 



Population 1 has the larger sample variance.



Degrees of freedom 15 and 15

Using *F* table, area in tail is between .05 and .10

Two–tail *p*–value is between .10 and .20

Exact *p*–value corresponding to *F* = 2.35 is .1087

*p*–value > .10, do not reject *H*0. Cannot conclude that there is a difference between the population variances.

34. 





Degrees of freedom 30 and 24

Using *F* table, area in tail is between .025 and .05

Two–tail *p*–value is between .05 and .10

Exact *p*–value corresponding to *F* = 2.08 is .0695

*p*–value  .10, reject *H*0. Conclude that the population variances are not equal.